Find the fourier series for $f(x) = x^2$ defined on the interval $[-\pi, \pi]$.

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\, dx = \frac{2\pi^2}{3}$$

$$\int x^2 \cos nx\, dx = \frac{x^2}{n}\sin nx - \frac{2}{n}\int x \sin nx\, dx$$

$$= \frac{x^2}{n}\sin nx - \frac{2}{n}\left(\frac{-x\cos nx}{n} + \frac{1}{n}\int \cos nx\, dx\right)$$

$$= \frac{x^2 \sin nx}{n} + \frac{2x\cos nx}{n^2} - \frac{2}{n^3}\sin nx + C$$

$$\int_{-\pi}^{\pi} x^2 \cos nx\, dx = \frac{4\pi \cos n\pi}{n^2}$$

$$a_n = \frac{4(-1)^n}{n^2}$$

$$f(x) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2}$$

https://www.desmos.com/calculator/m3rbjvhn2g